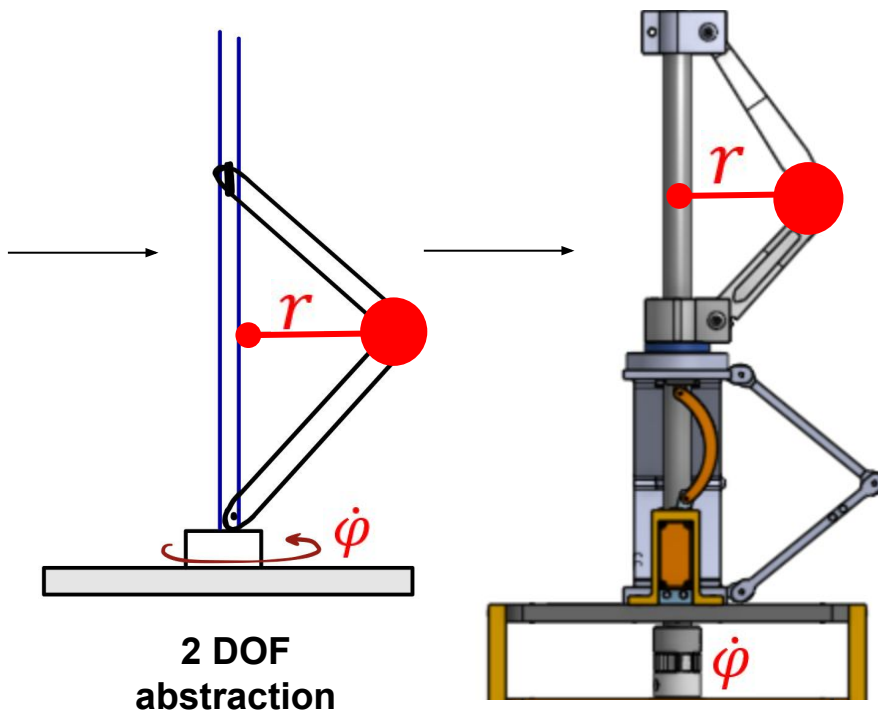
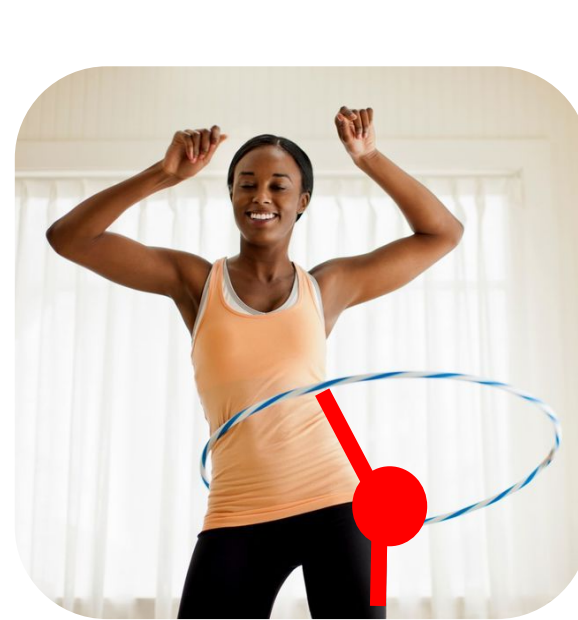


## Introduction

What are the principles behind humans hula hooping?



3 inertial coordinates:  
 $x, y, \varphi$

2 system parameters:  
 $R, \dot{\varphi}$

human

2 DOF abstraction

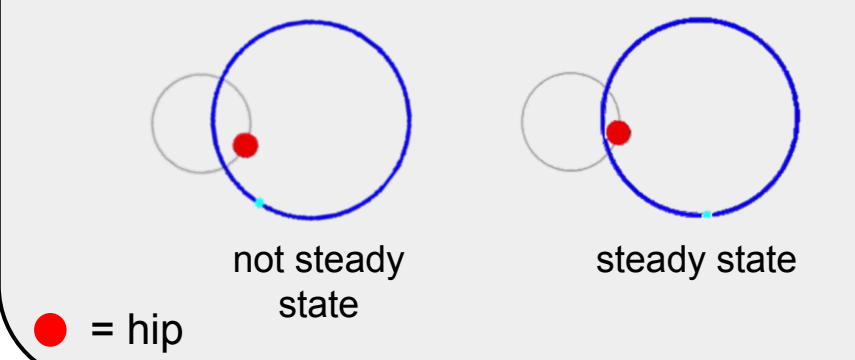
final system

How do we minimize *rise time*?

What is the optimal *phase difference*?

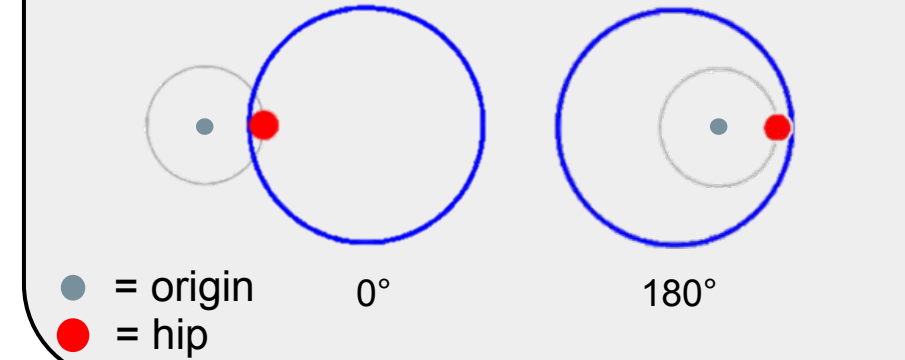
**rise time**

time it takes the hoop to reach steady state (rolling contact between hoop and "person")



**phase difference**

average angular difference between the hoop and person during steady state. e.g:



Hypotheses: **Humans use a spiral trajectory which may offer lower rise times. Higher radius and higher speed will generate quicker rise times.**

## Simulation Methods

Hip Trajectory

Simulation

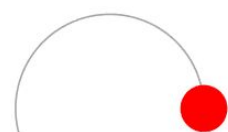
Hoop Trajectory

Vary  $R$  and  $\dot{\varphi}$  to generate hip trajectories...

Run forward simulation in MATLAB and apply spring-damper contact dynamics.

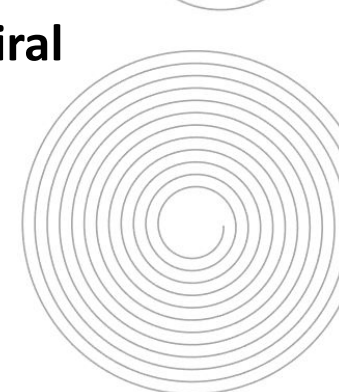
Track effect on hoop trajectory.

**Circular**



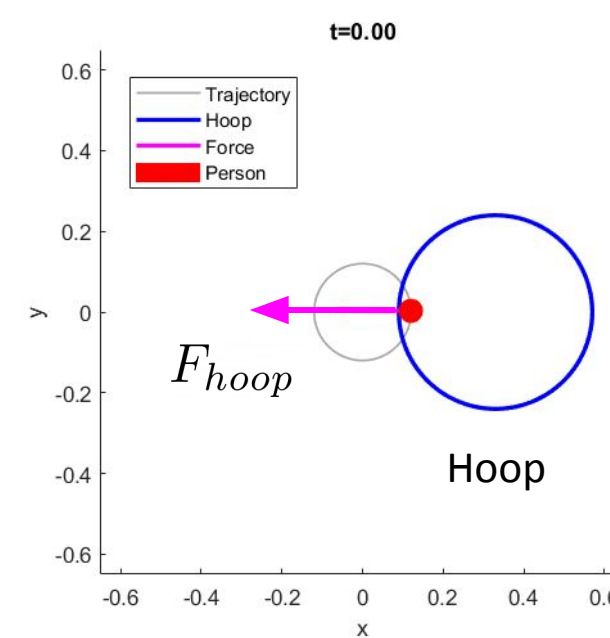
$$F_{hoop} = (Kx_{error} + D\dot{x}_{error})\hat{n}_r$$

**Spiral**



Apply force onto hoop in normal direction when collision occurs.

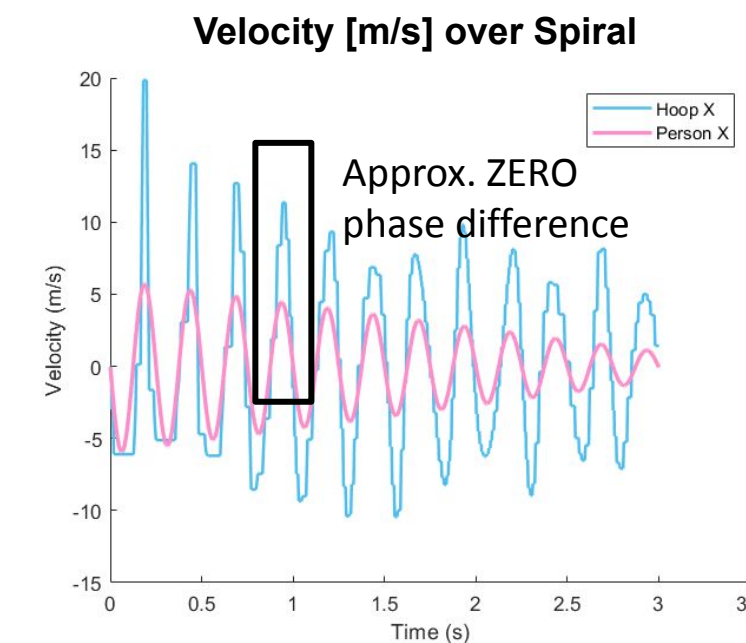
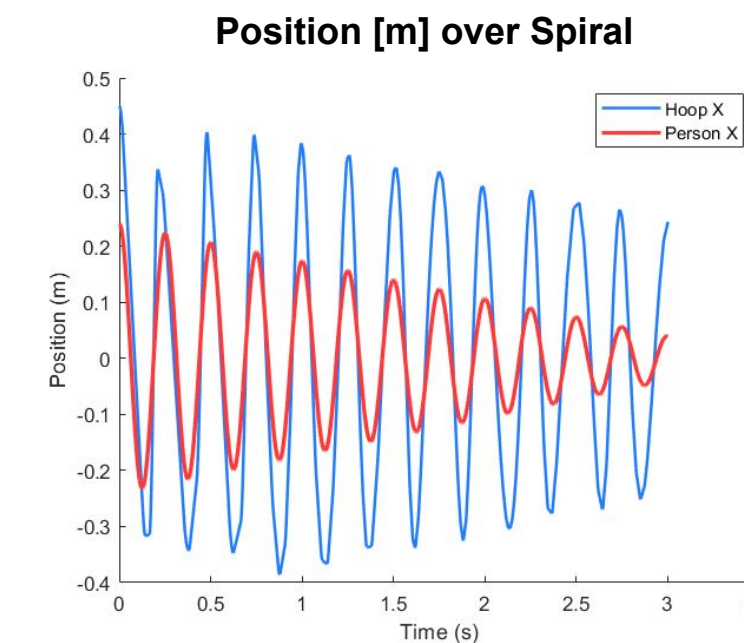
Compute **rise time** based on the moving average of contact force.



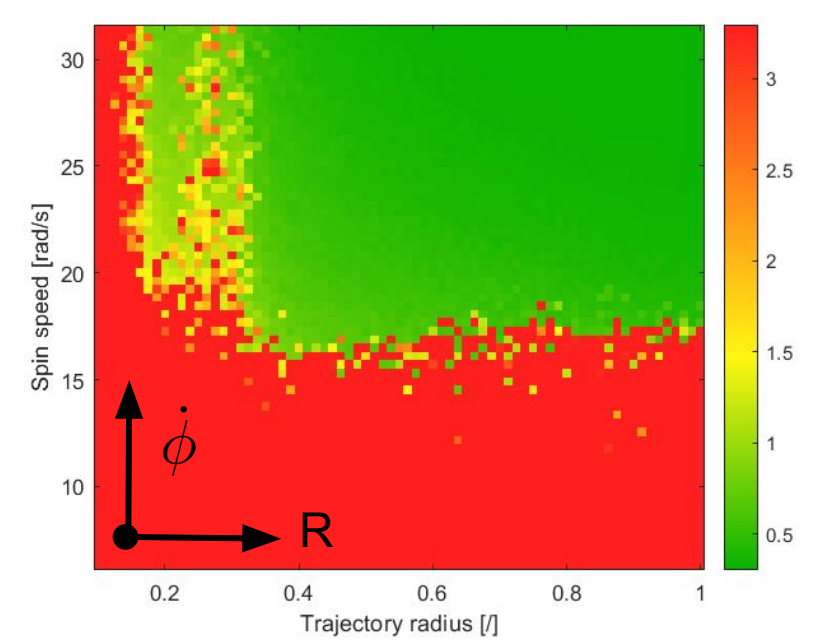
## Simulation Results

Track hoop movement in simulation to see how it reacts to given hip trajectory.

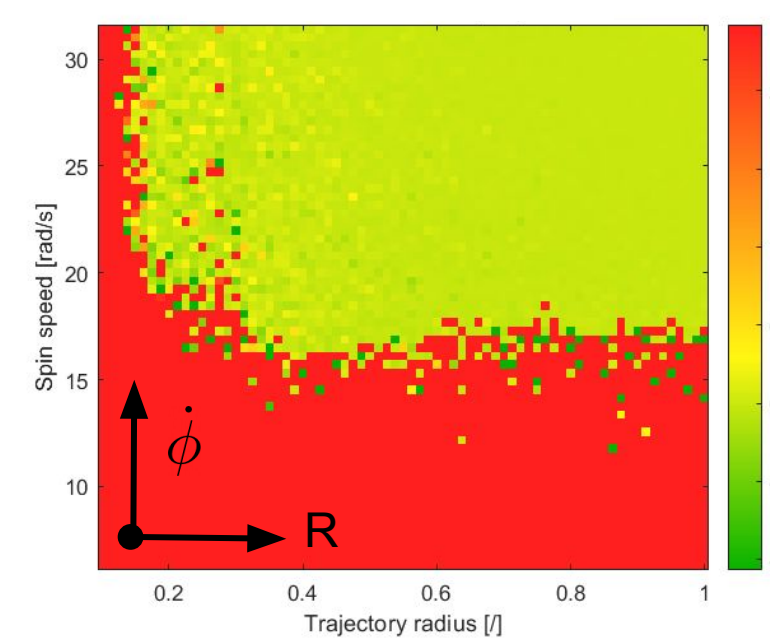
For given hip trajectory, speed and position of hoop are amplified.



Rise Time [s]



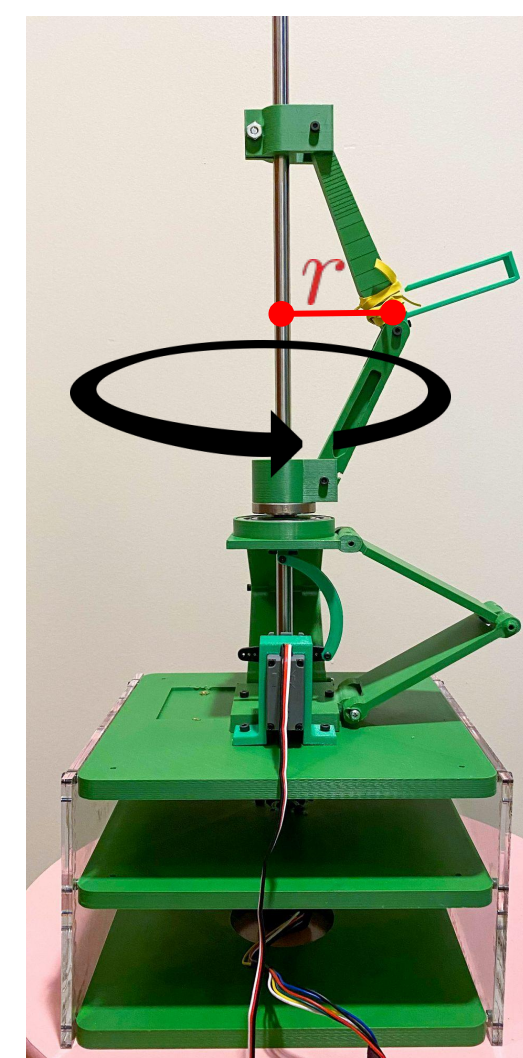
Phase Diff. [rad]



Confirmed that the best trajectories have zero phase difference

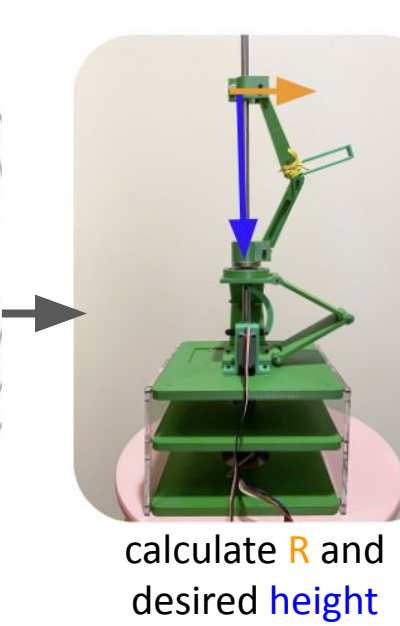
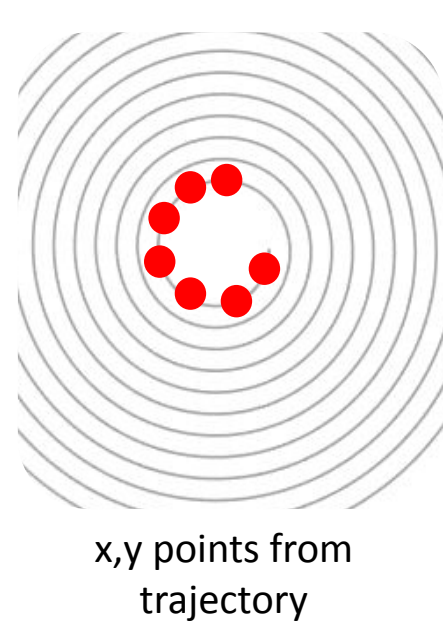
High speed and hip radius give lowest rise time. (maximal contact)

## Hardware Design



**2 DOF system**  
 $R$  - actuated by servo  
 $\varphi$  - actuated by motor

**servo control**



**motor control**

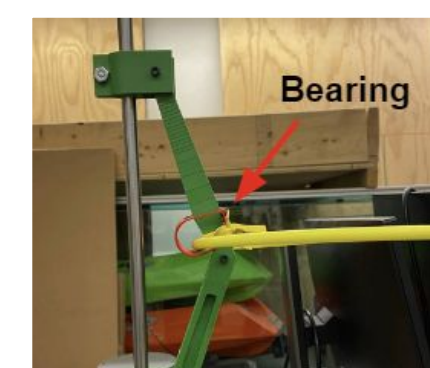
$$duty\_cycle = \frac{R * i_{des} + k_i * v + K_p * (i_{des} - i) + K_i * \sum (i_{des} - i)}{voltage}$$

simplified to no feedback + commanded speed

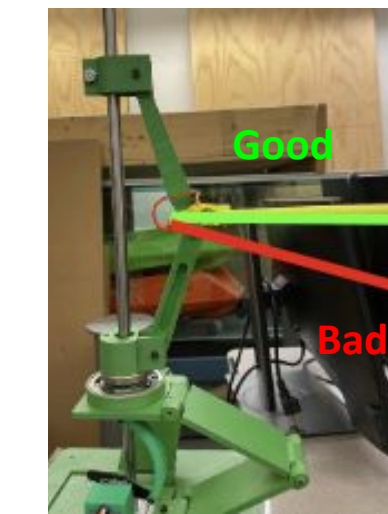
**hoop attachment method**

**Bearing Purpose:**  
Constrain the hoop while allowing it to spin

**Bearing Position:**  
Operate at phase difference of 0 (ideal config)



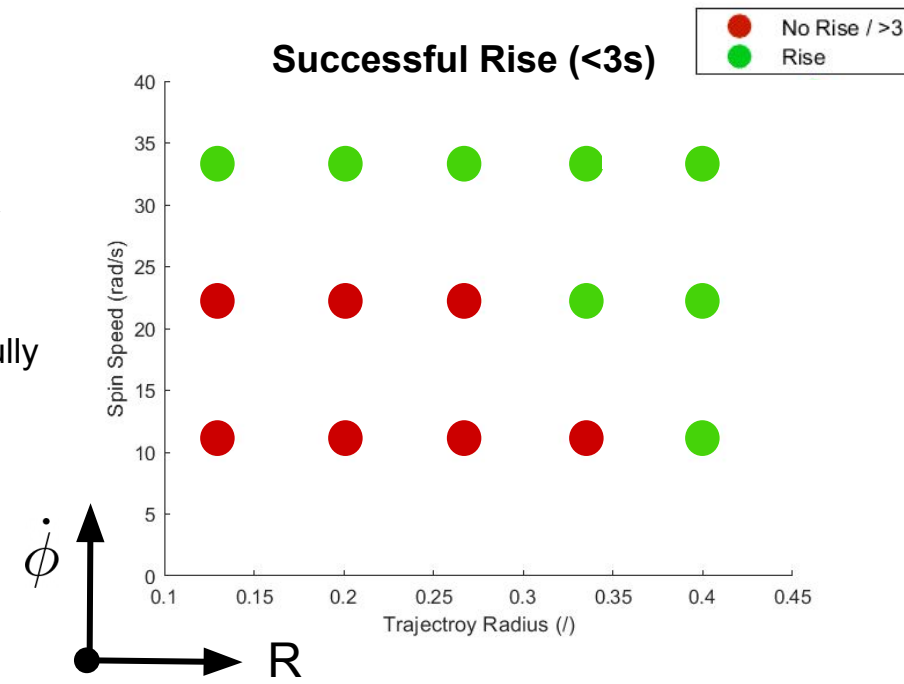
## Experimental Results



Measured by analyzing video recording of trial.

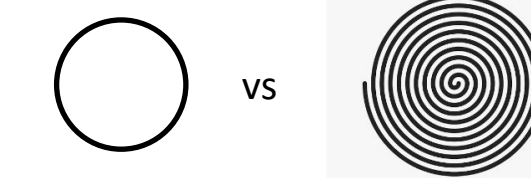
**Rise Time Experiment**

1. Pick trajectory radius.
2. Try 3 different duty cycles / angular speeds.
3. Note which trials successfully lift the hoop in 3 seconds.
4. Repeat for selected set of radii.



**Spiral Rise Time Comparison**

Ran two trials at the same speed for a spiral vs a circle at the middle spiral radius



Trajectory	Radius	Speed	Rise Time
Circle	0.3	11.1	Inf
Circle	0.3	22.1	0.65
Spiral	0.4 -> 0.2	22.1	0.4

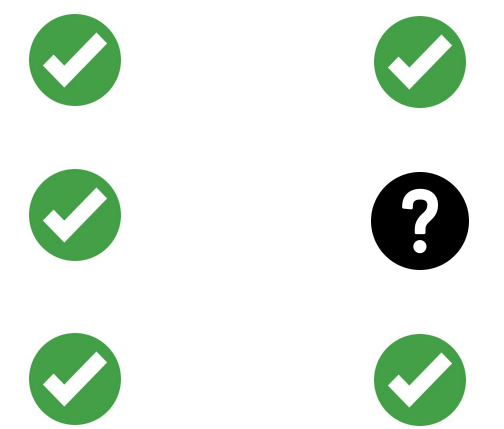
## Conclusions

Higher radius, higher speed gives minimum rise time to steady state.

Phase difference of 0 is optimal.

Spiral hip trajectory gives lower rise time than circular trajectory.

**Simulation**      **Hardware**



## Future Work

- Gather more data on trajectory radius and rise time in hardware, this time measuring time values instead of pass fail
- Create 3D Simulation in MATLAB, factoring in gravity
- Improve the accuracy of the robot model
  - Mimic the human waist (rolling contact) in hardware
  - Decrease friction at contact point

## References

[1] Cross, R. (2021). Physics of a hula hoop. In Physics Education (Vol. 56, Issue 2, p. 025015). IOP Publishing. <https://doi.org/10.1088/1361-6552/abd875>

Thank you to Prof. Kim, Andrew, Elijah, Adi, and Se Hwan for guiding us!